BACCALAURÉAT GÉNÉRAL ET TECHNOLOGIQUE
ÉPREUVE SPÉCIFIQUE DES SECTIONS EUROPÉENNES
MATHEMATIQUES – ANGLAIS

Corrigé 5 – Pythagoras and equations

Thème : Geometry

For the first part, the expected points are :
• Pythagoras's theorem and the link with Pythagorean triples.
• Use of the theorem in everyday life.
• Things about Pythagoras's life(13-knot-rope ; 1st written proof ; Pythagoras's students...).
• Perhaps things about Diophantus's life ? (Greek mathematician ; maybe talk about Diophantine equations (equations having only whole numbers for coefficients and having only whole numbers as acceptable solutions))
(• Other important theorems in geometry, like the intercept theorem for example)

Exercise

1. a. Let x be the length DC. Then $AC = x + 1$.
In the right-angled triangle ADC we can use Pythagoras's theorem : $AC^2 = AD^2 + DC^2$.
$(x+1)^2 = 7^2 + x^2$. So $2x = 48$ and $x = 24$. Finally : DC = 24 and AC = 25.

b. (7,24,25) is a Pythagorean triple because 7 ; 24 and 25 are three positive integers and $7^2 + 24^2 = 25^2$.

2. a. F is the midpoint of the line segment [DC], so DF = 12. AB = 12 because ABFD is a rectangle (quadrilateral with four right angles).
Area (ABCD) = $\frac{12+24}{2} \times 7 = 126$. The area of the polygon ABCD is 126 in².

b. $12 \times 7 = 84$. The area of ABFD is 84 in².
$\frac{Area\ (ABFD)}{Area\ (ABCD)} = \frac{84}{126} = \frac{2}{3} \approx 0.667$. ABFD's area is approximately 66.7% of the total area.

3. a. (AD) // (EF) and F is the midpoint of [DC]. Using a particular case of the intercept theorem, we can say that E is the midpoint of [AC] and that $EF = \frac{AD}{2} = \frac{7}{2} = 3.5$.
So : $AE = \frac{AC}{2} = \frac{25}{2} = 12.5$.
Perimeter (AEFD) = AE + EF + FD + DA = 12.5 + 3.5 + 12 + 7 = 35.
The perimeter of the polygon AEFD is 35 in.

b. $35 \times 2.54 = 88.9$. The perimeter of the polygon AEFD is 88.9 cm.

Bonus : • Prove that if you choose 1 and 2, you will find the Pythagorean triple (3,4,5) with Diophantus's method.
• Prove Diophantus's method that if you take two numbers m and n, then $(2mn, m^2 - n^2, m^2 + n^2)$ is a Pythagorean triple.
$(2mn)^2 + (\mathrm{m}^2 - \mathrm{n}^2)^2 = 4m^2n^2 + \mathrm{m}^4 - 2\mathrm{m}^2\mathrm{n}^2 + \mathrm{n}^4 = \mathrm{m}^4 + 2\mathrm{m}^2\mathrm{n}^2 + \mathrm{n}^4 = (\mathrm{m}^2 + \mathrm{n}^2)^2$.
• Prove that if (a, b, c) is a triple, then so is (ka, kb, kc) for any natural number k.